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QUANTUM THEORY IN KNOWLEDGE REPRESENTATION

A Novel Approach to Reasoning with a Quantum Model of Concepts

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Abstract

This thesis explores novel approaches to enhance the reasoning capabilities of AI systems by leveraging the mathematical framework of quantum theory. While quantum theory initially emerged as a description of subatomic particle behaviour, recent perspectives have recognised its peculiarities as advantageous features applicable beyond the realm of physics. We aim to harness the mathematical foundations of quantum theory as a *general probabilistic theory*, extending its potential to knowledge representation and reasoning, discrete modelling, cognition, machine learning, and game theory. The interdisciplinary approach interweaves insights from quantum foundations, quantum information theory, category theory, linear algebra, probability theory, cognitive science, and artificial intelligence. Starting from the paradigm of *quantum picturialism* proposed by Coecke and Kissinger (2018), which offers a diagrammatic language rooted in category theory, to represent and reason about quantum theory in a compositional way, the thesis demonstrates that the underlying structure of quantum theory is not limited to physical systems, but holds relevance across diverse systems and processes.

The unique features of quantum theory, including compositionality, contextuality, and complementarity, induce enhanced modelling capabilities in various domains. This work takes a *utilitarian* modelling approach, claiming that while the processes under study may not be inherently quantum, the shared mathematical structures provide a quantum *modelling advantage*. Given the established use of linear algebra and probability theory in classical artificial intelligence, the expressiveness of quantum theory in these domains is a natural extension. In a literature survey, we link together various applications across domains, from quantum game theory and quantum satisfiability to quantum machine learning, quantum natural language processing and quantum cognition. Moreover, such quantum models naturally lead to quantum algorithms, native to quantum computers, which can be leveraged to obtain a quantum *computational advantage*

The longstanding challenge of formally representing and reasoning with concepts, which are integral to cognitive processes such as perception, reasoning, and language, has sparked significant interest in both cognitive science and artificial intelligence. The study delves into the *quantum model of concepts* as proposed by Tull et al. (2023), which combines conceptual space theory, introduced by Gärdenfors (2000), with quantum theory. The diagrammatic language serves as a compositional framework for both theories, unearthing their common structure and showcasing how the quantum conceptual model, unlike its classical counterparts, exploits quantum theory's distinctive features for enhanced modelling capabilities. This connection facilitates the translation of insights between the two domains, laying the groundwork for the realisation of a quantum conceptual model on existing quantum hardware.

By operationalising the model as a hybrid quantum-classical architecture, we explore the extent to which quantum conceptual models can serve as a practical intermediate representation for artificial neuro-symbolic agents that combine both symbolic and subsymbolic reasoning. We address the *symbol grounding problem* of how such agents might acquire the semantics of new quantum concepts and show that quantum conceptual models and their concepts can be learned automatically from raw data through a combination of self-supervised and supervised learning techniques (Figure 1).

However, in order to take this model beyond mere concept recognition and truly *reason* with concepts, we capitalise on the relationship between the quantum conceptual model and symbolic AI approaches, given by the compositionality of quantum picturialism. We show how the flexibility of this compositional framework allows us not only to ground complex concepts indirectly in terms of simpler ones in a way that is interpretable to humans, but also makes it possible to transform concepts into general generative processes. Finally, all these properties of quantum conceptual models culminate in an operational conceptual model that is capable of reasoning with abstract concepts and perceptual uncertainty to effectively solve visual relational *blackbird* puzzles, similar to Raven's Progressive Matrices, which are commonly used to test human intelligence (Figure 2).

The model's flawless 100% accuracy in solving all *blackbird* puzzles emphasises the significance of learning meaningful compositional concept representations, combined with both symbolic and subsymbolic reasoning. Quantum conceptual models are the result of an amalgamation of ideas from different fields of research, each with its own set of open questions, and as such, this work opens up many avenues for future research.

I would like to thank my supervisor Prof. Dr. Dr. Geraint Wiggins for the opportunity to work freely on this truly fascinating subject and for his guidance throughout the process.

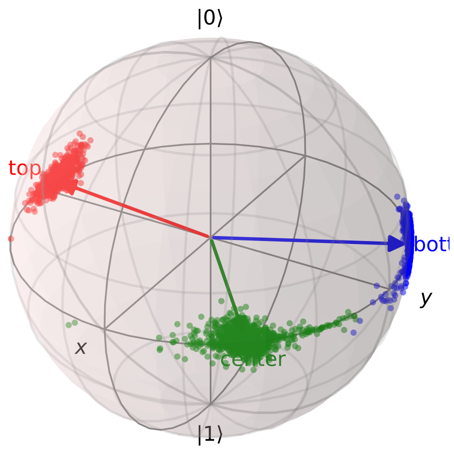

Figure 1: Bloch sphere visualisations of some of the simplest quantum concepts that can be learned using a single qubit. On the domains of *Position* (left) and *Size* (right), the concepts of {top, centre, bottom} (left) and {small, medium, large} are represented as vectors, and their instances as points of the same colour. While different concepts are spread out over the sphere, the corresponding instances are centred closely around their concepts. The structure inherent to these domains is retained in the quantum representations, with instances of related concepts blending into each other.

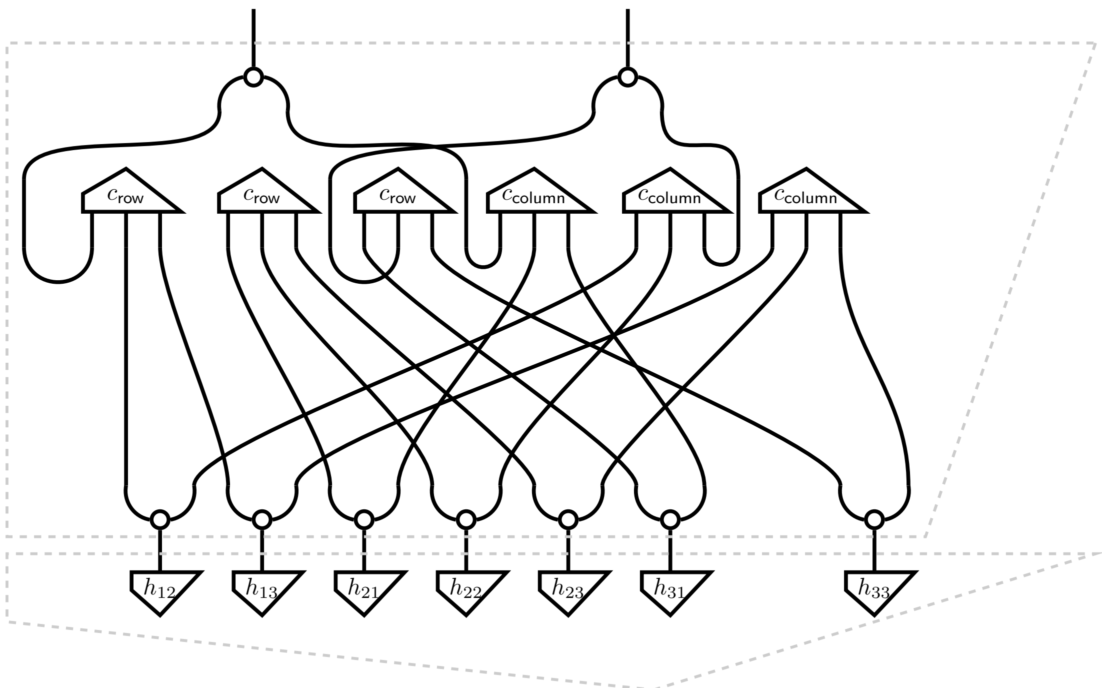

Figure 2: Diagrammatic representation of the quantum circuit implementing a generative composite conceptual process that can solve *blackbird* puzzles.

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