A hybrid local search algorithm for the Continuous Energy-Constrained Scheduling Problem

Roel Brouwer[0000−0001−8300−4290], Marjan van den Akker[0000−0002−7114−0655], and Han Hoogeveen[0000−0001−8544−8848]

Department of Information and Computing Sciences, Utrecht University, Princetonplein 5, 3584 CC, Utrecht, The Netherlands
{r.j.j.brouwer, j.m.vandenakker,j.a.hoogeveen}@uu.nl

Keywords: continuous scheduling · resource-constrained scheduling · mixed-integer linear programming · simulated annealing

This abstract is based on a paper by [2], which has been accepted for publication in Journal of Scheduling. In this paper, we present a hybrid local search approach for a scheduling problem with flexible resource requirements. This type of problem often occurs in practice in the domain of planning energy consumption. As far as we are aware, we are the first to propose an algorithm that can find near-optimal solutions for reasonably sized problems.

In scheduling problems with resource constraints, jobs are often assumed to have a fixed duration and are not allowed to change the amount of resource they consume over time. In certain applications, however, these assumptions are too limiting. Typically, such cases can be found in areas where a (cumulative) amount of work (or resource) is required to complete a job, but the rate of consumption is not necessarily fixed. An important example is given by energy related applications such as demand-response in electricity consumption and electric vehicle charging. Suppose the charging of electric vehicles in a certain neighborhood is coordinated by a so-called Aggregator (see for example [9]). Each arriving vehicle has a given amount of power that it wants to charge and a desired departure time. A vehicle can be charged at different rates and this rate may change over time. However, at each point in time the total amount that we charge is limited by for example the network capacity or the capacity of the transformer connecting the neighborhood to the mid-voltage grid. We want to make a schedule where we assign a charging profile to each vehicle, such that we respect the capacity limits and each vehicle can leave on time, or if the latter is not feasible the total delay is minimal.

To find schedules that can accommodate flexible execution profiles, we consider an extension of the traditional scheduling problem, previously introduced as the Continuous Energy-Constrained Scheduling Problem (CECSP) by [4].

This problem is described as follows. A set of n jobs has to be processed on a continuous resource R. This means that, at any time, multiple jobs can be processed simultaneously and at different rates, as long as their total consumption does not exceed the amount of available resource, denoted by P. Each job j ($j = 1, \dots, n$) requires a total amount of resource equal to E_j. The goal is to find a schedule in which each job j is executed in the period starting with its release time r_j and ending with its deadline $\bar{d}_j$. Preemption is not allowed, and therefore, from its start until its completion, each job must consume resources at a rate of at least P_j^- and at most P_j^+ units. The objective is to minimize the total weighted completion time. We look at the case where both the resource and time are continuous.

The CECSP was originally introduced as a generalization of the Cumulative Scheduling Problem (CuSP), which was formulated by [1] as a subproblem of the Resource Constrained Project Scheduling Problem (RCPSP). The RCPSP is a very general problem concerning the scheduling of activities subject to precedence, time and resource constraints (see for example the surveys by [7] and [8]). The CuSP corresponds to the special case of the RCPSP without precedence constraints and with only one resource. The CECSP generalizes the CuSP as the resource capacity requirement is considered to be a range with a lower and an upper bound, rather than a fixed value, and the consumption rate can vary during the execution of the job. As a result, the processing time is no longer fixed, but depends on the consumption rate during its execution. Through its relation to CuSP, [5] proved that the feasibility problem of CECSP is $\mathcal{NP}$-hard. [6] provided a hybrid branch-and-bound algorithm and mixed-integer linear program to find exact solutions for small instances.

We consider a slightly modified version of the problem originally introduced by [4]. The main difference is that we do not consider efficiency functions (but our approach can be extended to deal with these), which implies that the total amount of resource (energy) used for any feasible solution is equal. Hence, minimizing the total amount of energy used, which is the objective considered by [4], makes no sense then, and therefore we consider the traditional objective of minimizing the total weighted completion time, which corresponds to minimizing the total weighted delay.

We show that any given feasible schedule for our problem can be transformed into a feasible schedule with equal objective value in which the resource consumption profile per job is constant between any two *events*, which correspond to starting or completing a job. These events form a crucial element in our approach. We show that the $\mathcal{NP}$-hardness of our problem stems from the existence of the lower bounds P_j^-. We show that, if we know for each interval which jobs are being processed, then we can solve our problem as a Linear Program, since we then know for each job whether the lower bound on its consumption rate is equal to P_j^- or 0. Our solution approach is based on this result. We use a *hybrid local search approach*, using simulated annealing and linear programming, to solve instances of the CECSP. In the first stage we specify the *order* of the $2n$ events corresponding to the start and completion of the n jobs; this leads to $2n - 1$ intervals. In the subproblem we use Linear Programming to compute the optimal values for the times at which these $2n$ events take place, taking the ordering into account, together with the consumption rate for each job for each interval. We then modify the event order using Simulated Annealing, etc.

As far as we are aware, we are the first to propose such a decomposition and investigate a combination of local search and mathematical programming techniques for CECSP. In addition, we provide a mixed-integer linear programming (MILP) formulation and compare the performance of our hybrid approach to it in terms of quality and runtime. Our hybrid local search approach matches the MILP formulation in solution quality for small instances ($n \leq 10$), and is able to find a feasible solution for larger instances ($15 \leq n \leq 50$) in reasonable time. This approach opens the door for finding solutions to larger instances.

Moreover, we found that the feasibility problem is solvable in polynomial time if we drop the lower bounds on resource consumption from the problem. We formulate a flow-based solution algorithm for that case. We use this algorithm to select challenging problem instances for a new benchmark set of problem instances. Our approach is evaluated on these instances.

Finally, in a follow-up paper [3] we show that our approach can be applied to a myriad of related (energy-constrained) scheduling problems with some minor modifications.

References

1. Baptiste P., Le Pape C., and Nuijten W. (1999). Satisfiability tests and time-bound adjustments for cumulative scheduling problems. *Annals of Operations Research 92*, pp. 305–333. https://doi.org/10.1023/A:1018995000688
2. Brouwer, R., Van Den Akker, M., and Hoogeveen, H. (2023). A hybrid local search algorithm for the Continuous Energy-Constrained Scheduling Problem. Accepted for publication in *Journal of Scheduling*. Preprint at Arxiv: https://arxiv.org/abs/2311.16177
3. Brouwer, R., Van Den Akker, M., and Hoogeveen, H. (2024). A hybrid optimization framework for the General Continuous Energy-Constrained Scheduling Problem. Preprint at Arxiv: https://arxiv.org/abs/2403.03039

4. Nattaf, M., Artigues, C., and Lopez, P. (2014). A polynomial satisfiability test using energetic reasoning for energy-constraint scheduling. In: *International Conference on Project Management and Scheduling (PMS 2014)*, pp. 169–172. https://hal.archives-ouvertes.fr/hal-00978706
5. Nattaf, M., Artigues, C., Lopez, P., and Rivreau, D. (2016). Energetic reasoning and mixed integer linear programming for scheduling with a continuous resource and linear efficiency functions. *OR Spectrum 38*, pp. 459–492
https://doi.org/10.1007/S00291-015-0423-X.
6. Nattaf, M., Artigues, C., and Lopez, P. (2017). Cumulative scheduling with variable task profiles and concave piecewise linear processing rate functions. *Constraints 22*, pp. 530–547
https://doi.org/10.1007/S10601-017-9271-4
7. Hartmann, S., and Briskorn, D. (2010). A survey of variants and extensions of the resource-constrained project scheduling problem. *European Journal of Operational Research 207*, pp. 1–14
https://doi.org/10.1016/j.ejor.2009.11.005
8. Hartmann, S., and Briskorn, D. (2022). An updated survey of variants and extensions of the resource-constrained project scheduling problem. *European Journal of Operational Research 297*, pp. 1–14 https://doi.org/10.1016/j.ejor.2021.05.004
9. Sadeghian, O., Oshnoei, A., Mohammadi-ivatloo, B., Vahidinasab, V., and Anvari-Moghaddam, A., (2022). A comprehensive review on electric vehicles smart charging: Solutions, strategies, technologies, and challenges, *Journal of Energy Storage*, 54, 105241, https://doi.org/10.1016/j.est.2022.105241.